«Archi-gineer» form-finding tool for tensile structures: interweaving of the design process with structural optimization for reduced workflow and time saving.

Noémie Aureau

Abstract. *Membrane structures provide remarkable architectural possibilities thanks to their ability to cover very large spans with elegance, lightness and very few materials in new or existing buildings. However, a unique shape will be stable and structurally feasible for a tensile canopy depending on its boundary conditions. How to free the architect's imagination while ensuring the shape of his design to be buildable? The tool presented in this paper combines parametric modeling and structural analysis and allows for a variety of design options to be instantaneously explored both quantitatively and qualitatively by adjusting various parameters. As the most impactful decisions in the design process are made at the start of any project, this tool is of great help to the design team by assisting good decision-making at this early start.*

1 Introduction

- Collaboration between architects and engineers: form-finding & form optimization at early stage.

A close collaboration between architects and engineers is the way creating new shapes for architecture that are elegant, beautiful, comfortable and ergonomic but that are also sustainable, energy efficient, structurally optimized, cleverly buildable and materially economical.

During the architectural competition for a railway station in France, Setec was asked to propose and calculate different options for a modular canopy with anticlastic shape modules. As these type of forms are best suited for tensile structures and to foster this rewarding "archi-gineer" work, we developed a Grasshopper form-finding component including Visual Basics scripts enabling an architect to design a structurally optimized tensile structure or a cable net directly through Rhinoceros 3D.

- Tensile structures & form finding: why is it crucial?

If most structures can be optimized to be structurally efficient, buildable or environmental efficient after the design stage, this design process is not possible with tensile structures. Typically for this type of structure, shape and forces are intrinsically interwoven into the formal design. The initial process of finding the optimal shape for a given configuration is commonly referred to as "form-finding". This form-finding process has to be performed at the very beginning of the design stage.

2 Why is form-finding essential for tensile structures design?

Tensile structures provide remarkable architectural possibilities thanks to their ability to cover long spans with elegance, lightness and structural efficiency. Their design differs from the one of more conventional structures because the shape of the surface is determined according to the pre-tension in the mesh. Tensile membrane structures are made of flexible non-rigid materials. The geometry of the form in its initial state is very different from the loaded geometry including pre-stress in the mesh and wind or snow loads.

As a result, the key issue with these structures lies in the definition of their structural form. The geometry of the surface is required to meet the equations of equilibrium and ensure a distribution of the stresses in the cables that is as uniform as possible. It is also essential to know the stresses induced by the tensile structure in its edge ring insofar as its geometry and section directly depend on these stresses.

Standard “free” form design, the usual approach in architecture, cannot be applied with with this type of structure.

3 Scripting of the force density method with Grasshopper and Visual Basic.

3.1 The different calculation methods commonly used for form-finding.

Before the 70s, soap films models were used to find the optimal form for tensile structures. Today, the most commonly used numerical methods to perform form-finding are the dynamic relaxation method and the force density method.

3.2 Why using the force density method for tensile structures form-finding?

The force density method was developed in 1971 to facilitate the design of cable net roofs for Munich Olympic Stadium by Linkwitz and Schek together with Frei Otto and the InstitutfürleichteFlächentragwerke.

This method uses a surface that is describes as a system of nodes linked by branches. It can therefore be used to model either a tensile membrane structure or a cable net. Indeed we consider that a pre-stressed net or a tensile membrane structure are assumed to be describes as a pin-joint net of individual bars subjected to tension forces, which implies that there is no bending and no sagging of the individual elements due to dead loads.

The core concept behind the method relies in solving the equilibrium equations at each node of the mesh for a given set of boundary conditions: the coordinates of nodes on the edge ring of the tensile structure and the pre-stress applied to the structure. Due to the linearization of the matrix system, this method offers a remarkable rapid calculation allowing the form-finding process to be directly performed within the design software Rhinoceros and its graphical algorithm editor Grasshopper.

3.3 The force density method on an example.

First the process can be illustrated with a simple case of a single node linking four cables. In the diagram below, nodes 2-5 are fixed and their coordinates are known. These are the boundary conditions.

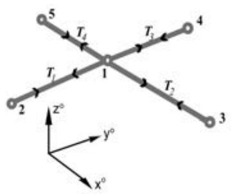


Figure 1: Single node linked to four fixed points.

If we write the equation of equilibrium of the central node in each direction x, y, z by introducing the tensions T_i in the cables, we obtain:

$$\frac{T_1(x_2-x_1)}{L_1}+\frac{T_2(x_3-x_1)}{L_2}+\frac{T_3(x_4-x_1)}{L_3}+\frac{T_4(x_5-x_1)}{L_4}=f_{1x}$$
$$\frac{T_1(y_2-y_1)}{L_1}+\frac{T_2(y_3-y_1)}{L_2}+\frac{T_3(y_4-y_1)}{L_3}+\frac{T_4(y_5-y_1)}{L_4}=f_{1y}$$
$$\frac{T_1(z_2-z_1)}{L_1}+\frac{T_2(z_3-z_1)}{L_2}+\frac{T_3(z_4-z_1)}{L_3}+\frac{T_4(z_5-z_1)}{L_4}=f_{1z}$$

In the previous system of equations, the right members are x-, y- and z-components of the force applied to the node 1. In the form-finding process for a pre-stressed system, the contributions of external loads are taken into account for the fixed nodes on the surface edge only. The respective lengths of the elements L_i are nonlinear functions of end coordinates at the ends of the cables.

The "trick" of the force density method is to introduce constant values for the force-length ratios, q_i, also called "force densities": $q_i = \frac{T_i}{L_i}$.

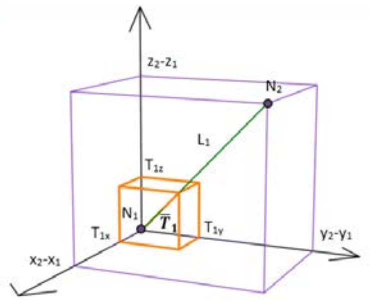


Figure 2: Geometrical connections between two nodes.

By introducing the force densities, the system becomes linear:

$$q_1(x_2 - x_1) + q_2(x_3 - x_1) + q_3(x_4 - x_1) + q_4(x_5 - x_1) = 0$$
$$q_1(y_2 - y_1) + q_2(y_3 - y_1) + q_3(y_4 - y_1) + q_4(y_5 - y_1) = 0$$
$$q_1(z_2 - z_1) + q_2(z_3 - z_1) + q_3(z_4 - z_1) + q_4(z_5 - z_1) = 0$$

Knowing the values of qi, the coordinates of the central free node can be easily deduced:

$$x_1 = \frac{q_1 x_2 + q_2 x_3 + q_3 x_4 + q_4 x_5}{q_1 + q_2 + q_3 + q_4}$$

Coordinates along y and z can be deduced similarly.

3.4 Matrix formulation.

Generally for a branch m connected to nodes k and g, the L_{mx} , L_{my} and L_{mz} components of vector $\{L_m\}$ can be expressed using the connectivity matrix [C] and the coordinates of the end nodes by:

$$L_{mx} = x_k - x_g = [1 \quad -1]\begin{Bmatrix} x_k \\ x_g \end{Bmatrix} = [C]\{X\}$$
$$L_{my} = y_k - y_g = [1 \quad -1]\begin{Bmatrix} y_k \\ y_g \end{Bmatrix} = [C]\{Y\}$$
$$L_{mx} = z_k - z_g = [1 \quad -1]\begin{Bmatrix} z_k \\ z_g \end{Bmatrix} = [C]\{Z\}$$

Based on the previous example, we can write:

$$\begin{Bmatrix} L_{1x} \\ L_{2x} \\ L_{3x} \\ L_{4x} \end{Bmatrix} = [C]\{X\} = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{Bmatrix} = \begin{Bmatrix} x_2 - x_1 \\ x_3 - x_1 \\ x_4 - x_1 \\ x_5 - x_1 \end{Bmatrix}$$

The connectivity matrix [C] is the “branch-node” matrix of the reticulated system. To make its writing easy, its columns are ordered as follows: the first column corresponds to the free node (the central one) while the following columns correspond to the fixed nodes (the one on the edge of the surface). The system can be written as follows:

$$\{L_x\} = [C]\{X\}$$
$$\{L_y\} = [C]\{Y\}$$
$$\{L_z\} = [C]\{Z\}$$

x-, y- and z-components of the internal force in cable m, P_{mx}, P_{my} and P_{mz} can be respectively expressed as the product of force density q_m and $\{L_m\}$ as follows:

$$\{P_{mx}\} = [Q]\{L_{mx}\}$$
$$\{P_{my}\} = [Q]\{L_{my}\}$$
$$\{P_{mz}\} = [Q]\{L_{mz}\}$$

where [Q] is the square diagonal matrix of force densities:

$$[\mathrm{Q}] = \begin{bmatrix} \mathrm{q}_1 & . & . & . \\ . & \mathrm{q}_2 & . & . \\ . & . & \mathrm{q}_3 & . \\ . & . & . & \mathrm{q}_4 \end{bmatrix}$$

The equilibrium of forces along x direction becomes:

$$[C]^T[Q][C]\{X\} = \{P_x\}$$

or:

$$\begin{bmatrix} -1 & -1 & -1 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} q_1 & . & . & . \\ . & q_2 & . & . \\ . & . & q_3 & . \\ . & . & . & q_4 \end{bmatrix} \begin{Bmatrix} x_2 - x_1 \\ x_3 - x_1 \\ x_4 - x_1 \\ x_5 - x_1 \end{Bmatrix} = \begin{Bmatrix} P_{1x} \\ P_{2x} \\ P_{3x} \\ P_{4x} \\ P_{5x} \end{Bmatrix}$$

In the above equation, P_{1x} is equal to 0 while the remaining P_{ix} are equal to reactions at fixed nodes along x. As a result, we obtain:

$$[C]^T[Q][C]\{X\} = \begin{bmatrix} -q_1 & -q_2 & -q_3 & -q_4 \\ q_1 & 0 & 0 & 0 \\ 0 & q_2 & 0 & 0 \\ 0 & 0 & q_3 & 0 \\ 0 & 0 & 0 & q_4 \end{bmatrix} \begin{Bmatrix} x_2 - x_1 \\ x_3 - x_1 \\ x_4 - x_1 \\ x_5 - x_1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ P_{2x} \\ P_{3x} \\ P_{4x} \\ P_{5x} \end{Bmatrix}$$

We therefore find the previous result for the expression of the x-coordinate of the free node:

$$x_1 = \frac{q_1 x_2 + q_2 x_3 + q_3 x_4 + q_4 x_5}{q_1 + q_2 + q_3 + q_4}$$

with the remaining equations:

$$q_1(x_2 - x_1) = P_{2x}\ ;\ q_2(x_3 - x_1) = P_{3x}\ ;\ q_3(x_4 - x_1)P_{4x}\ ;\ q_2(x_5 - x_1) = P_{5x}$$

Coordinates along y and z can be deduced similarly.

4 The different steps of the algorithm in the Grasshopper script.

Now we will describe how the force density method was used to create a Grasshopper script to perform form-finding directly with Rhinoceros.

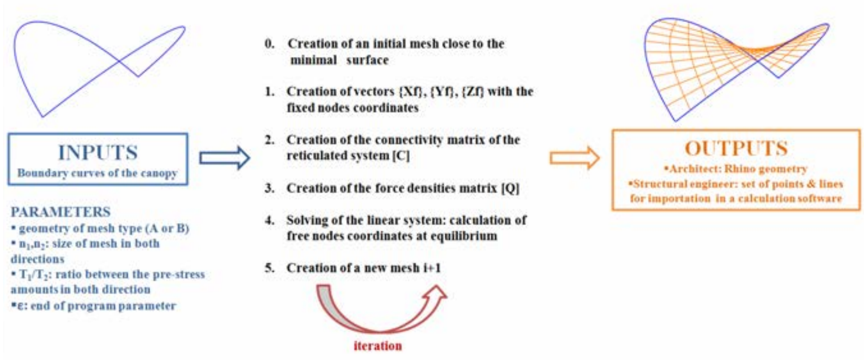


Figure 3: View of the Grasshopper program.

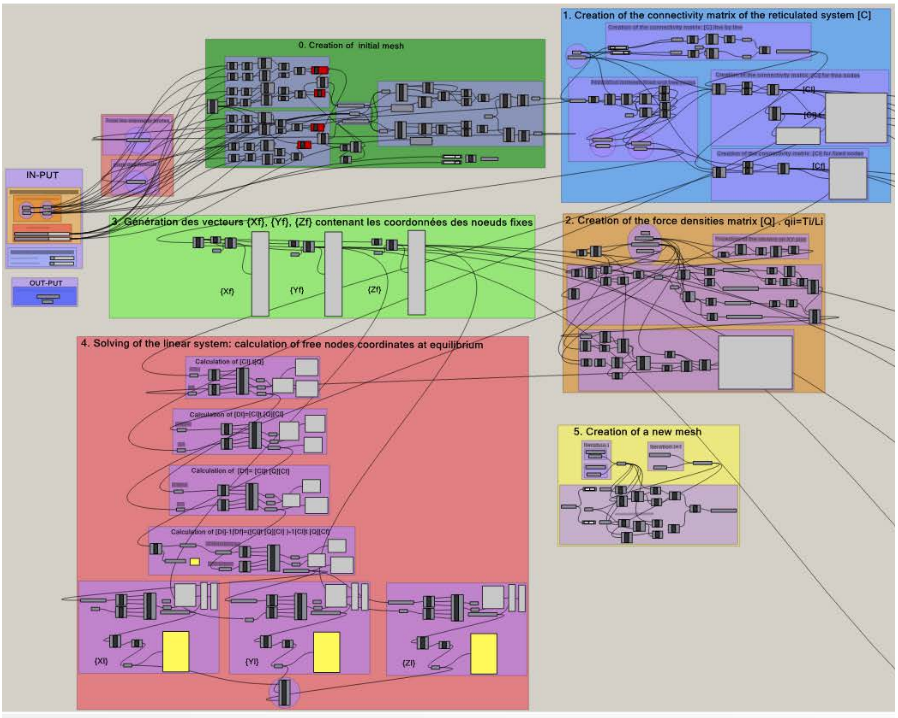


Figure 4: View of the Grasshopper program.

4.1 Input.

1. Selection of the contour curves of the tensile structure directly in the 3D design software Rhinoceros.
2. Setting of mesh size parameters n and n' and geometry of mesh (type A or B)
3. Setting of the pre-stress in the cables T_1 and T_2in the two main directions.
4. Ɛ, end of program parameter.

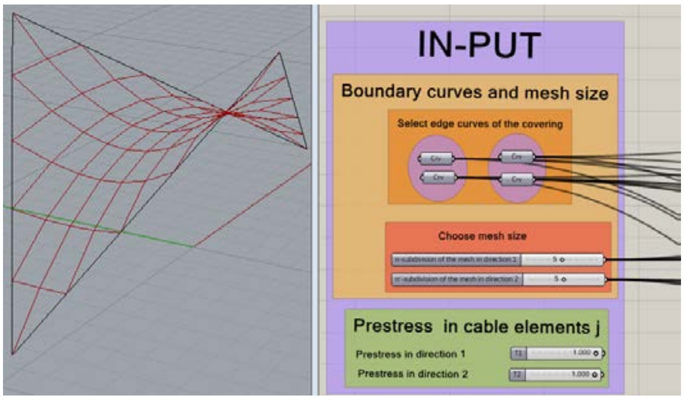


Figure 5: Left. Form-found mesh in Rhinoceros 3D calculated with the Grasshopper form-finding component. Right. In-put box in the Grasshopper script.

4.2 Creation of an initial mesh close to the minimal surface.

Once the parameters are set by the designer, the program builds an initial mesh close to the minimal surface to ensure the algorithm to converge without too many iteration. This mesh is used to calculate the initial lengths of the branches and to deduce the set of force densities $q = \frac{T}{L}$ for the first iteration of the algorithm.

The program then creates the 3 vectors {Xf}, {Yf}, {Zf} with the fixed nodes coordinates.

4.3 Creation of the connectivity matrix of the reticulated system [C] and creation of the force densities matrix [Q].

Based on the previous results the connectivity matrix has as many rows as branches in the mesh and as many columns as nodes. The matrix [C] is regular and is filled row by row. For a row corresponding to branch m whose ends are respectively k and g (k <g), all terms of the row are equal to 0, except:

$$c_{mk} = -1$$
$$c_{mg} = 1$$

The matrix is then divided in two parts: $[C_{lx}]$ and $[C_{fx}]$, corresponding to the subset of free nodes and to the subset of fixed nodes.

To create the force densities matrix, the program calculates and multiplies two vectors:

b$_2$rows corresponding to the b$_2$ branches in direction 2

b$_1$rows corresponding to the b$_1$ branches indirection 1

$$\begin{pmatrix} \frac{1}{L_0} \\ \vdots \\ \frac{1}{L_{b_2-1}} \\ \frac{1}{L_{b_2}} \\ \vdots \\ \frac{1}{L_{b_2+b1-1}} \end{pmatrix}$$

and

b$_2$rows corresponding to the b$_2$ branches in direction 2

b$_1$rows corresponding to the b$_1$ branches indirection 1

$$\begin{pmatrix} 1 \\ \vdots \\ 1 \\ \frac{T_1}{T_2} \\ \vdots \\ \frac{T_1}{T_2} \end{pmatrix}$$

where $\frac{T_1}{T_2}$ is the ratio between pre-stress T_1 and T_2 along the two main directions of the mesh defined by the designers as an input of the form finding process.

b$_2$rows corresponding to the b$_2$branches in direction 2

b$_1$rows corresponding to the b$_1$ branches in direction 1

$$\begin{bmatrix} \frac{1}{L_0} & 0 & 0 & & & \\ 0 & \ddots & 0 & & 0 & \\ 0 & 0 & \frac{1}{L_{b_2-1}} & & & \\ & & & \frac{T_1}{T_2 L_{b_2}} & 0 & 0 \\ & 0 & & 0 & \ddots & 0 \\ & & & 0 & 0 & \frac{T_1}{T_2 L_{b_2+b1-1}} \end{bmatrix}$$

At every new iteration i, a new matrix of force densities [Q] is calculated using the branches lengths of i-1 iteration$(L_m^0)_{i-1}$.

Once the square diagonal matrix of force densities [Q] has been calculated, another box of the program solves the linearized system.

4.4 Solving of the linear system: calculation of free nodes coordinates at equilibrium.

We can write for the n nodes of the mesh:

$$[C]^T[Q][C]\{X\} = \{f_x\} \quad (*)$$

The vectors $\{X\}$and$\{f_x\}$are respectively the x-coordinates of the n nodes and the external loads along x at each node. If we only consider the n_l equilibrium equations corresponding to the n_l free nodes, we obtain:

$$[C_{lx}]^T[Q][C_{lx}]\{X_l\} = \{f_{lx}\} - [C_{lx}]^T[Q][C_{fx}]\{X_f\}$$

If we introduce $[D_x]$and $[D_{fx}]$defined as follows:

$$[D_x] = [C_{lx}]^T[Q][C_{lx}]$$
$$[D_{fx}] = [C_{lx}]^T[Q][C_{fx}]$$

(*) can be re- written as:

$$[D_x]\{X_l\} = \{f_{lx}\} - [D_{fx}]\{X_f\}$$

Similar equations can also be established along y and z:

$$[D_x] = [D_y] = [D_z] \text{ and } [D_{fx}] = [D_{fy}] = [D_{fz}]$$

Moreover, in the case of form-finding for a tensile structure, external loads applied to free nodes are not taken into account for the search for initial shape.So the system simplifies in:

$$[D_x{}^0]\{X_l\} = -[D_{fx}{}^0]\{X_f\}$$

Where $[D^0] = [C_l]^T[Q^0][C_l]$and $[D_f{}^0] = [C_l]^T[Q^0][C_f]$.

By judiciously choosing the coefficients q_j, this approach provides the structurally optimal surface for a given set of boundary conditions with a very limited number of iterations.

4.5 Iterations and output.

After having solved the linear system at iteration i, a new mesh is created from the cloud of nodes to perform the iteration i+1.Aftereach iteration, the following ratio is calculated for each node k:

$$d_k = \frac{|\overline{X_\iota - X_{\iota-1}}|}{\min\left(\frac{L}{n};\frac{L}{n'}\right)}$$

Where L is the maximum distance between two points on the edge curves of the tensile surface and n and n' are the mesh size parameters chosen by the designer. Using a new Grasshopper component called Hoopsnake that allows the writing of recursive loops, the program stops if the following condition is verified (see figure 12):

$$d = \max\left(\frac{|\overline{X_\iota - X_{\iota-1}}|}{\min\left(\frac{L}{n};\frac{L}{n'}\right)}\right) < \varepsilon$$

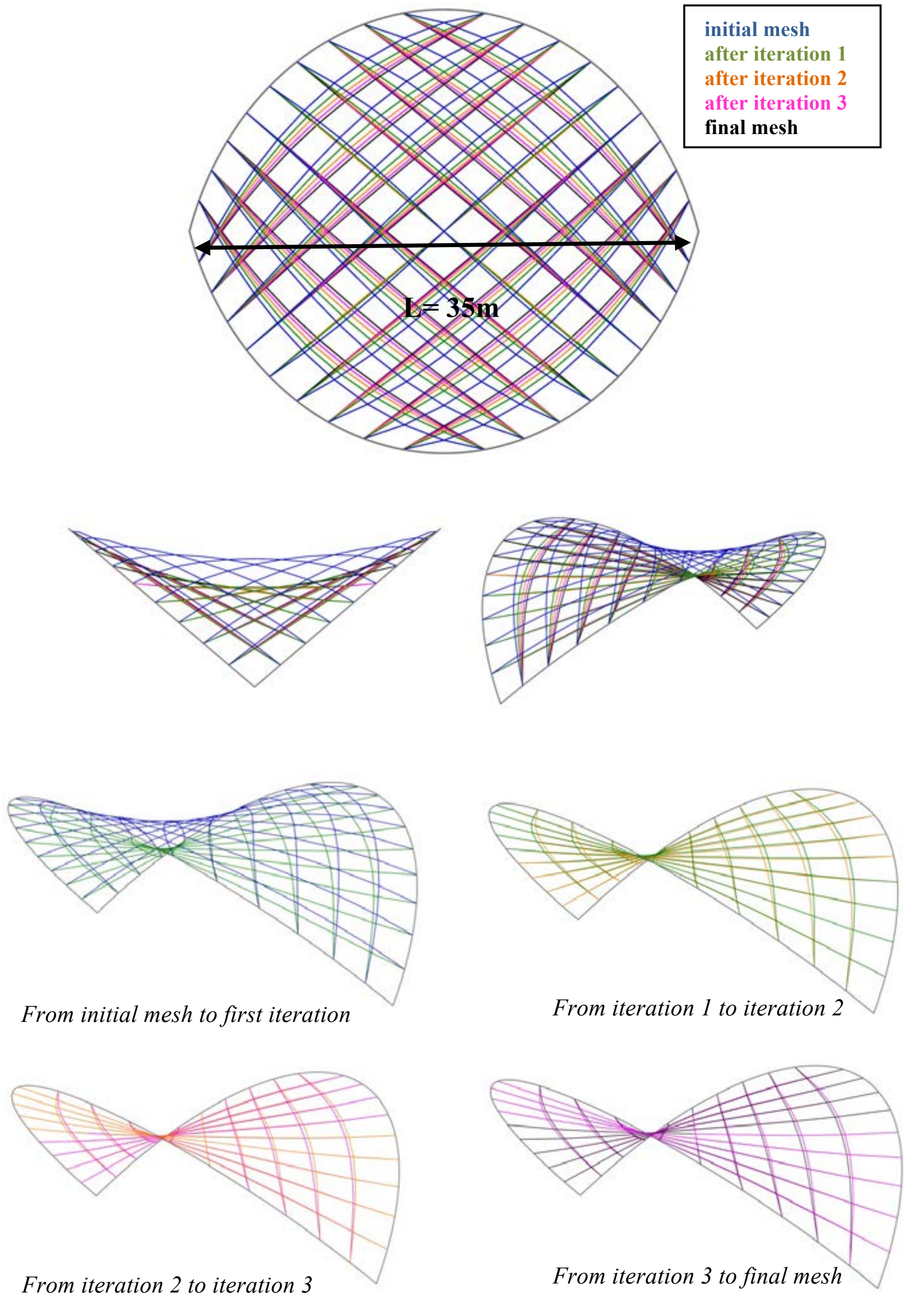


Figure 11: Convergence and mesh evolution after 4 iterations of the loop.

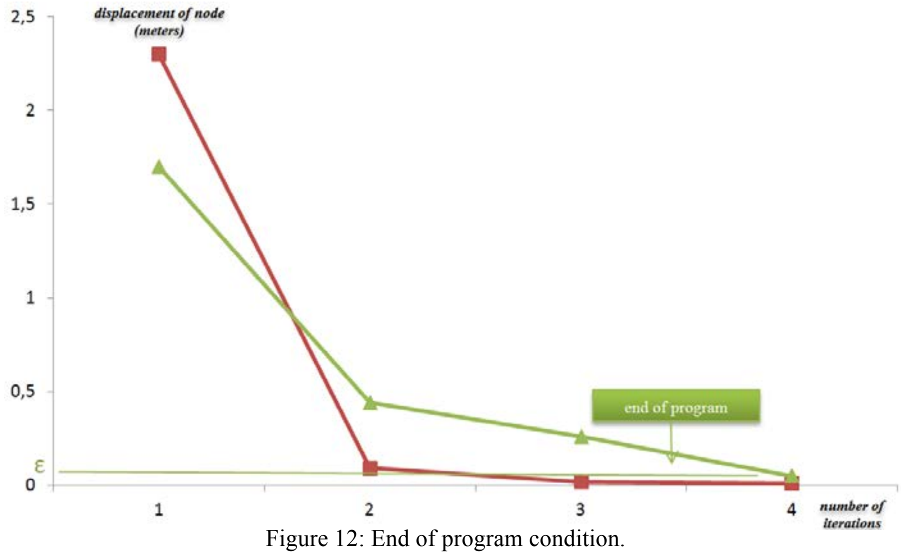


Figure 12: End of program condition.

In most cases, 4 to 6 iterations are enough to get very close to the structural optimum. The user sets the tolerance ε. In the example below, L= 35m and ε =5 cm. The programs stopped after 4 iterations only.

5 A user-friendly interface for time saving and reduced workflow.

5.1 Structural optimization within the design software.

This form-finding tool provides a real-time performance feedback to assist at the early stage of design. In less than one minute (a few seconds can be enough depending on the size of the mesh and the resulting number of nodes), the architect is able to find the equilibrium geometry of the tensile structure for a given set of parameters of his choice. This generates a productive dialogue between the architect and the structural engineer. If some structural engineering softwares like GSA Analysis also allow performing form-finding for tensile structures or cable nets, the advantage of the Grasshopper form-finding component is to offer valuable time saving by avoiding workflows between the design and the calculation software.
One of the strong benefits in using a parametric model is that it can also explore a wide range of possible designs very quickly.

5.2 Parameters of choice and degree of freedom for the architect.

The geometrical shapes of pre-stressed and hanging structures can be interpreted and consequently be described as 'figures of equilibrium'.

Depending on the position of the exterior anchoring points – at high masts and at low fixed fundaments-, the type of meshes and their orientation, and the amount of pre-stress induced in each direction, an unlimited number of different shapes can be generated.But once the anchoring points are set, for example on the contour of the covering, and the amount of pre-stress in both main directions of the mesh is fixed, a unique shape ensures the net or the membrane to be in equilibrium in its initial state (without external loads). For one set contour, the form-finding tool offers the possibility for the architect to create the optimal shape for a tensile structure that meets his design intents by adjusting four different parameters:

- Boundary curves of the canopy (see figure 13): they can be closed or open depending on the architectural needs and the structural possibilities to anchor the canopy;
- Geometry of mesh: type A or B (see figure 14);
- Size of mesh (which can be different in the two main directions of the net;
- Ratio between the pre-stress amounts in the two main directions of the mesh (see figure 15).

Depending on cases, it will be more interesting to use one or another mesh type. For the design of a cable net for example, the choice can be made according to aesthetic preference but also according to structural reasons. Indeed the pre-stress needed in a cable is a function of its deflection: the lower the curvature of the cable is, the higher the pre-stress needed in the cable will be. This is why the first option is preferred if the supporting structure cannot bear too much pre-stress: in figure 14, options B will allow the cables to work more efficiently with less pre-stress in the net.

The program also introduces the possibility to modify the ratio between the pre-stress amounts in the two main directions of the mesh for two reasons. On one hand it gives an additional degree of freedom for the architect to choose the curvatures of the tensile surface that fit his design intents the most (see figure 15). On the other hand, it is also interesting to have a different pres-stress in the two main directions to avoid a resonance phenomenon of the cable net during the construction stage. Last but not least, it can be cases where the bearing structure can be more loaded in one direction than the other, and then it can be crucial to be able to adjust T_1 and T_2. For instance it can be the case if the architect wants to cover an existing building's yard: some anchoring points might be less resistant in the old structure. The pre-stress value also depends on external loads intensity. Basically cables should remain in tension under any external load case. If the cable net structure is subject to a stronger external load case in one particular direction (descendant for example), it can be interested to adjust the roof geometry, so to adjust pre-stress, in order to be able to bear the strong external load without oversizing the whole structure.

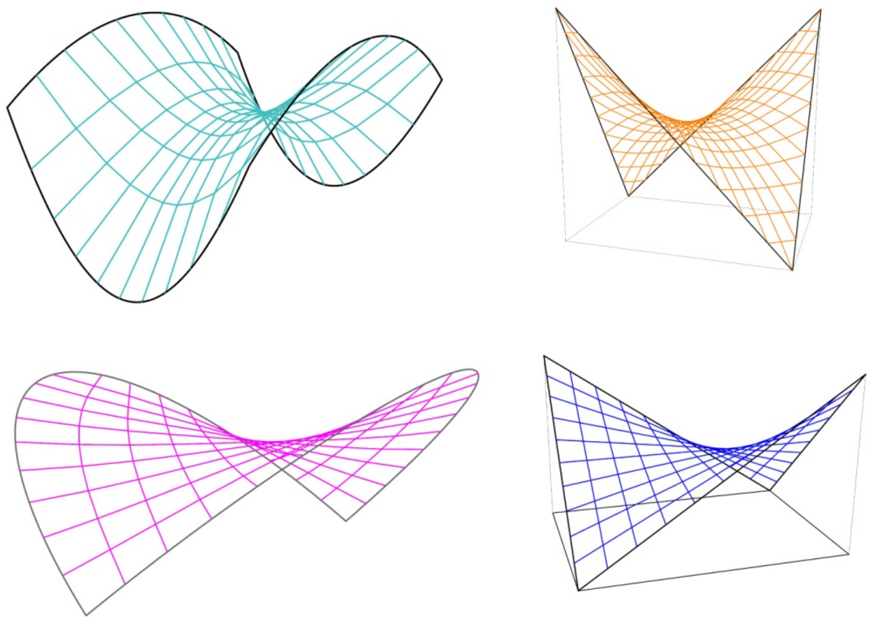
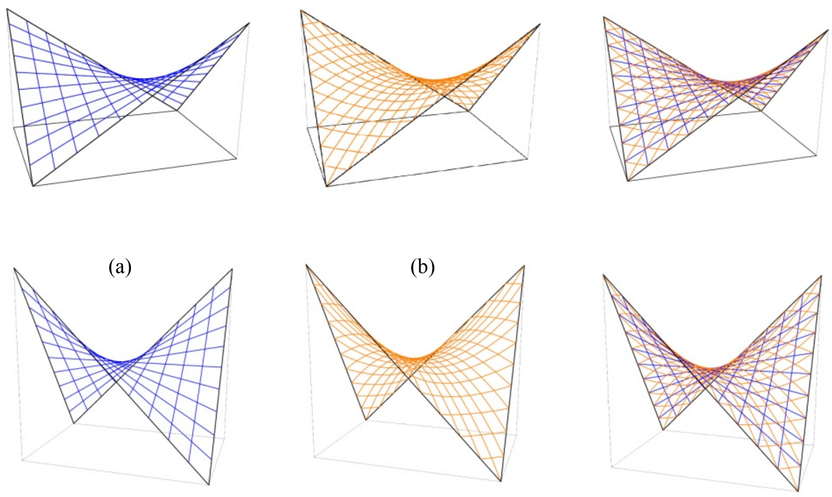


Figure 13: Examples of possible geometries with different boundary curves.

(a) (b)



(a) (b)

Figure 14: Examples of possible mesh geometries: (a) option A; (b) option B.

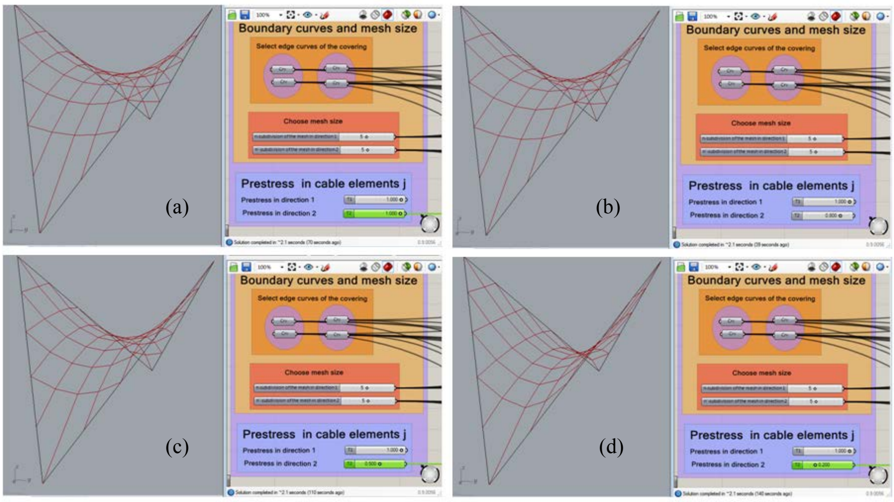


Figure 15: Evolution of the form-found geometry for different sets of pres-stress: (a) $\{T_1;\ T_1\} = \{1;\ 1\}$;(b) $\{T_1;\ T_1\} = \{1;\ 0.8\}$; (c) $\{T_1;\ T_1\} = \{1;\ 0.5\}$; (d) $\{T_1;\ T_1\} = \{1;\ 0.2\}$.

By allowing both the designer and the engineer to play with previous parameters, the tool enables both criteria of structural optimization and aesthetics to be met at the same time.

5.3 Useful output both for the architect and for the structural engineer.

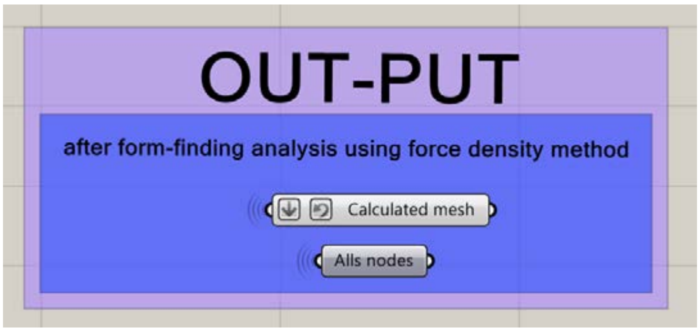


Figure 16: View of the "Output box".

- For the designer: Rhino 3D geometry

- For the structural engineer:

  1. 3D set of points and lines for direct importation into an engineering software for structural analysis and sizing of cables section;
  2. Ratio between pre-stress loads in the two main directions to evaluate the minimal pre-stress needed to ensure admissible displacements of the mesh.
  3.

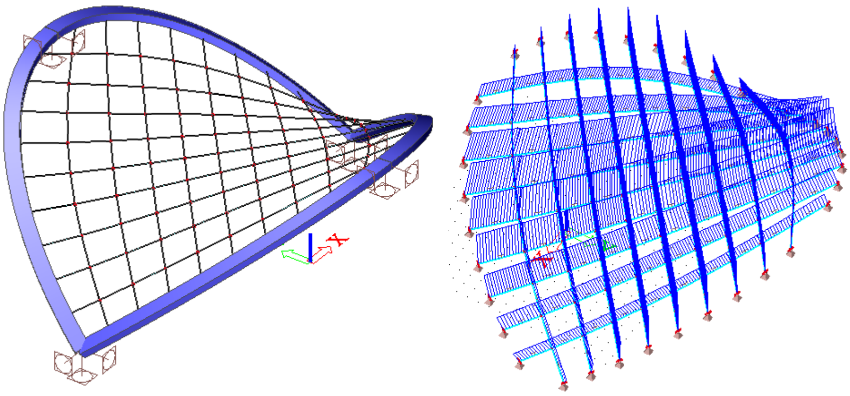


Figure 17: Left. Calculation model for structural analysis.Right. Tensile stresses in the net.

6 Conclusion.

With its high flexibility of use, this tool for tensile structure design is of remarkable help to both the designer and the engineer in order to generate optimized shapes in a playful way. It can be used to create a new wide-span canopy but can also to cover some parts of an existing building. In future projects, the tool could be easily improved by introducing the possibility for the user to add masts in the bearing structure, just as in the roof of the Olympic stadium in Munich. This could be realized by introducing the option to select fixed nodes within the initial mesh.

Once the net for a homogeneous distribution of constraints is obtained, another improvement would be to also add the possibility to optimize the net in order to have a regular size of mesh, allowing the latter to support standard panels.

Acknowledgements

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Author's address:

Noémie AUREAU (noemie.aureau@polytechnique.org)